Chapter 3 Percentage

GCE ‘O’ Level

Elementary Mathematics

Vocabulary List

- Percentage – 百分比
- Increase/Decrease – 增加/减少
- The lowest term – 最简形式
- Original – 原来的
- Profit and Loss – 利润和损失
- Discount – 折扣
- Goods and Services Tax – 消费税
- Commission – 佣金

Unit 01 Introduction to Percentage

1. A percentage is a fraction with a denominator of 100.

   The symbol % represents 'per cent'.

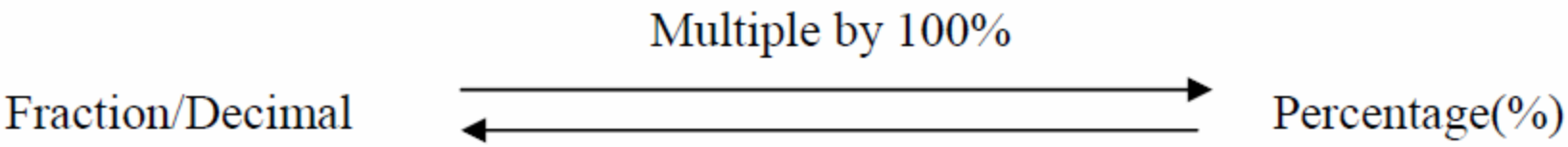
   e.g. If there are 20 red marbles in a jar of 100 marbles,
   we say that 20% of the marbles in the jar are red.

   In general, x percent is defined as x parts per hundred, i.e.

$$x\% = \frac{x}{100}$$

Unit 01 Introduction to Percentage

2. We can convert a fraction or decimal into a percentage and vice versa.



(a) drop the %.

(b) divide the number by 100.

(c) simplify and reduce to lowest terms

Unit 01 Introduction to Percentage

Example

There are 90 teachers in a school, of which 40 are male. Calculate the percentage of

(a) male teachers,

(b) female teachers,

in the school.

$(a)\ \frac{40}{90} \times 100\% = 44\frac{4}{9}\%$

$(b)\ 1 - 44\frac{4}{9}\% = 55\frac{5}{9}\%$

Unit 01 Introduction to Percentage

Example

(a) Express each of the following percentages as a decimal.

(i) 85% (ii) 228%

(b) Express each of the following decimals as a percentage.

(i) 0.16 (ii) 1.456

Solution:

(a)

(i) $85\% = \frac{85}{100} = 0.85$

(ii) $228\% = \frac{228}{100} = 2.28$

(b)

(i) $0.16 = (0.16 \times 100)\% = 16\%$

(ii) $1.456 = (1.456 \times 100)\% = 145.6\%$

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Unit 02 Percentage Change

The change in the value of an item can be expressed as a percentage increase or decrease in the original value.

An **increase** of 5% in the salary of a man who earns \$1600 per month that for every \$100 of the **original** salary, there is an increase of \$5, i.e. each \$100 in the original salary becomes \$105 in the **new** salary.

$$\therefore \frac{\text{New salary}}{\text{Original salary}} = \frac{105}{100}$$

$$\text{New salary} = \frac{105}{100} \times \text{Original salary}$$

$$= \frac{105}{100} \times \$1600$$

$$= \$1680$$

Unit 02 Percentage Change

On the other hand, a **decrease** of 5% in his salary means that for every \$100 of the **original** salary, there is a decrease of \$5, i.e. each \$100 in the original salary becomes \$95 in the **new** salary.

$$\therefore \frac{\text{New salary}}{\text{Original salary}} = \frac{95}{100}$$

$$\text{New salary} = \frac{95}{100} \times \text{Original salary}$$

$$= \frac{95}{100} \times \$1600$$

$$= \$1520$$

Unit 02 Percentage Change

In general,

New Value = final percentage × original value

Increase/Decrease = percentage increase/decrease × original value

Unit 02 Percentage Change

Example

The cost of a piece of furniture consists of the cost of wood at $300, the cost of paint at $200 and wages at $200. If the costs of wood and paint are increased by 12% and 7% respectively, while wages are decreased by 10%, calculate the percentage increase or decrease in the cost of the furniture.

Solution:

Original cost = \$300 + \$200 + \$200 = \$700

New cost = $\$300 \times (1 + 12\%) + \$200 \times (1 + 7\%) + \$200 \times (1 - 10\%)$
= \$730

so the percentage increased = $\frac{\$730 - \$700}{\$700} \times 100\% = 4\frac{2}{7}\%$

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Unit 03 Reverse Percentage

Example

After an increase of 5%, Shirley's monthly salary becomes $2205. Find her original monthly salary.

Solution:

Her original salary = $\$2205 \div (1 + 5\%)$

= $2100

Unit 04 Percentage in Practical Situation

- **Profit and Loss**
- **Discount**
- **Goods and Services Tax**
- **Commission**

Profit and Loss

A manufacturer produces goods at a certain cost. If the goods are sold at a price *higher* than the cost price, the manufacturer makes a **profit or gain.** However, if the manufacturer sells the goods at a price *lower* than the cost price, he suffers a **loss.**

Profit and Loss

Profit = Selling price – Cost price

Loss = Cost price – Selling price

We usually express the profit or loss as a percentage of the cost price:

$$\frac{\text{Profit}}{\text{Cost Price}} \times 100\% \text{ or } \frac{\text{Loss}}{\text{Cost Price}} \times 100\%$$

Profit and Loss

Example

Farhan buys 400 kg of bananas at $0.75 per kg. 10% of the bananas are rotten. Find the selling price per kg if he wants to earn a 80% profit on the cost price.

Solution:

Cost of bananas = $\$0.75 \times 400 = \$300$

Profit made = $\$300 \times 80\% = \$240$

Total sales = \$300 + \$240 = \$540

The unit selling price = $\frac{\$540}{400 \times (1-10\%)} = \$1.50/kg$

Discount

Very often, retailers are not able to sell defective merchandise, overstocked items or discontinued models at retail selling prices. To clear the merchandise in stock, the items are usually sold at a lower price, called the **sale price.** The difference between the original selling price, also known as the **marked price,** and the sale price is called the **discount.**

Discount

$$\text{Discount} = \text{marked price} - \text{sale price}$$

The discount is often given as a percentage of the marked price:

$$\frac{\text{Discount}}{\text{Marked Price}} \times 100\%$$

Discount

Example

A sculpture is sold for $533 after a discount of 18%.

(a) Calculate the marked price of the sculpture.

(b) If a 10% discount is given on the marked price of the sculpture before it is sold at a further discount of 8%, would the sale price still be $533? Show your working clearly.

Solution:

(a) The marked price = $\frac{\$533}{1-18\%} = \$650$

(b) the sale price

$= \$650 \times (1 - 10\%) \times (1 - 8\%)$

$= \$538.20$ which is more than \$533.

Goods and Services Tax

The various types of tax collected by the Singapore government go towards the funding of government expenditure such as national defense and education. Examples of taxes include **goods and services tax (GST)** and income tax. We will explore GST here and income tax in other chapter.

Goods and Services Tax

GST is paid in addition to the price of goods and services. GST is usually expressed as a certain percentage of the selling price. GST came into effect in Singapore in 1994, at 3%. It was increased to 4% in 2003, to 5% in 2004 and to **7%** in 2007.

Example

A piece of furniture costs $640 before GST. Assuming that GST is at 7%, calculate the total amount of money Raj has to pay for the piece of furniture.

Solution:

GST = $\$640 \times 7\% = \$44.80$

Total amount of money = $640 + $44.80

= $684.80

Percentage Point

The percentage **point** is defined as the *difference* between two percentages. For example, the increase from 5% to 7% is two percentage points, and not two percent.

Commission

- A commission is the payment an agent receives for selling or buying something on behalf of another party.
- It is usually given as a percentage of the cost price or the selling price.

Example

A property agent sells a house for $320 000, of which he receives a commission of 1.5%. Calculate the amount of commission the agent receives.

Solution:

The commission earned

$= \$320000 \times 1.5\%$

$= \$4800$